Chapter 20

RNN-ARAS Strategy for MCDM in Rough Neutrosophic Number Environment

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ABSTRACT

The purpose of the chapter is to extend the Additive Ratio Assessment (ARAS) strategy to the Rough Neutrosophic Number (RNN) environment which we call the RNN-ARAS strategy. The RNN is derived from the hybrid concept of rough set theories and single valued neutrosophic set theories. It comprehensively deals with uncertainty, incompleteness and inconsistent information. To demonstrate the applicability of the proposed strategy, an illustrative example of MADM problem is solved and the obtained ranking is compared with the rankings obtained from the existing strategies such as MABAC and COPRAS.

Keywords: Fuzzy set, neutrosophic set, single valued neutrosoophic set, rough set, rough neutrosopohic set, rough neutrosophic sets, ARAS, MCDM

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INTRODUCTION

Smarandache originated the notion of Neutrosophic Set (NS) [1] by extending the foundational notion of Fuzzy Set (FS) [2]) and Intuitionistic FS [3] to deal with uncertainty comprehensively. To tackle realistic decision problems, Single-Valued NSs [4] were introduced as a subclass of the NS. Several studies have provided detailed accounts of the theoretical enhancements and the varied applicability of NSs and SVNSs**.** [5-17]. Multi Criteria Decision Making (MCDM) and Multi Criteria Group Decision Making (MCGDM) are two areas in which SVNSs and their extensions and hybrid sets have been extensively explored as evidenced by the research publications [18–46]. Pawlak formulated Rough Set theory [47] as a tool for dealing with uncertain and incomplete information. Rough NSs (RNSs) were developed by Broumi et al. [48] via the fusion of RSs and SVNSs, aiming to deal with incompleteness and uncertainty. The main principles and applications of RNSs are comprehensively reviewed in [49, 50].

An MCDM strategy selects the best option from a set of feasible options while considering multiple conflicting criteria. In the Rough Neutrosophic Number (RNN) environment, a number of MCDM strategies have been proposed**.** Grey Relational Analysis (GRA) [51] was integrated with MCDM strategy in the RNN setting [52]. Several MCDM strategies were developed based on trigonometric Hamming similarity measures [53], in the RNN setting. TOPSIS

[54], the correlation-coefficient-measure-based MADM strategy [55], as well as the MADM strategies based on projection and bidirectional projection measures [56], were subsequently extended in the RNN settings.

Arithmetic and geometric mean operators based MCDM strategies were developed in [57]. MABAC [58] and COPRAS [59] were presented by Pramanik and Smarandache developed in the RNN setting.

The ARAS method for MCDM problems was introduced by Zavadskas and Turskis [60] . Subsequently, in a probability multi-valued NS environment, Liu and Cheng [61] proposed the ARAS approach. ARAS was extended in the SVNS environment by Adalı et al. [62] Mallick and Pramanik [63] proposed the ARAS strategy for MCGDM under the trapezoidal NS setting. Sahoo et al. [64] developed MCGDM framework for information retrieval considering popularity ranking.

Research gap: To date, no studies have proposed the ARAS strategy in the RNN setting.

Motivation: The identified research gap motivates us to develop the ARAS method.

Objectives: To present the ARAS strategy in the RNN environment which we name the RNN-ARAS strategy.

The rest of the paper is presented as follows. Preliminaries of the SVNSs and RNSs were presented in Background Section. RNN-ARASC strategy is developed in the next section. This is followed by solving a numerical example of an MADM problem using the RNN-ARAS strategy. Finally, the paper concludes by stating the future research scopes.

BACKGROUND

Definition 1. An SVNS [4] H in a universal set Θ is characterized by a truth Membership Function (MF) $\tau_{\Theta}(\theta)$, an indeterminacy-MF $\upsilon_{\Theta}(\theta)$, and a falsity-MF $\phi_{\Theta}(\theta)$ with $\tau_{\mathrm{H}}(\theta), \upsilon_{H}(\theta), \phi_{H}(\theta) \in [0,1], \forall \theta \in \Theta$.

When Θ is continuous, an SNVS H can be presented as:

$$H = \int_{\omega} \left\langle \tau_{\mathrm{H}}(\theta), \upsilon_{H}(\theta), \phi_{H}(\theta) \right\rangle / \theta, \forall \theta \in \Theta$$

and when Θ is discrete, an SVNS H can be presented as:

$$\mathrm{H} = \sum \left\langle \tau_{\mathrm{H}}(\theta), \upsilon_{H}(\theta), \phi_{H}(\theta) \right\rangle / \theta, \forall \theta \in \Theta$$

with $\tau_{\mathrm{H}}(\theta), \upsilon_{H}(\theta), \phi_{H}(\theta) \in [0,1]$, and $0 \le \sup \tau_{\mathrm{H}}(\theta) + \sup \upsilon_{H}(\theta) + \sup \phi_{H}(\theta) \le 3, \forall \theta \in \Theta$

The triplet $\tau_{\mathrm{H}}(\theta), \upsilon_{H}(\theta), \phi_{H}(\theta)$ is termed as the Single-Valued Neutrosophic Number (SVNN) and presented as $(\tau_{\mathrm{H}}, \upsilon_{H}, \phi_{H})$.

Let $e_1 = (x_1, \beta_1, y_1)$ and $e_2 = (x_2, \beta_2, y_2)$ be any two SVNNs with $x_1, \beta_1, y_1, x_2, \beta_2, y_2 \in [0,1]$, $(x_1 + \beta_1 + y_1) \in [0,3]$ and $(x_2 + \beta_2 + y_2) \in [0,3]$

Then, some selected operations involving SVNNs [4] are stated as follows;

i. $e_1 \oplus e_2 = (x_1 + x_2 - x_1 x_2, \beta_1, \beta_2, y_1 y_2)$ [Summation] (1)

ii. $e_1 \otimes e_2 = (x_1 x_2, \beta_1 + \beta_2 - \beta_1 \beta_2, y_1 + y_2 - y_1 y_2)$ [Multiplication] (2)

iii. $\lambda e_1 = (1-(1-x_1)^{\lambda}, \beta_1^{\lambda}, y_1^{\lambda})$, $\lambda > 0$ [Scalar multiplication] (3)

iv. $e_1^{\lambda} = (x_1^{\lambda}, 1-(1-\beta_1)^{\lambda}, 1-(1-y_1)^{\lambda})), \lambda > 0$ (4)

2. Euclidean distance function. Euclidean distance [23] between $e_1=(x_1,\beta_1,y_1)$ and $e_2=(x_2,\beta_2,y_2)$ is defined as:

$$\delta=\left[\frac{1}{3}\left\{(x_1-x_2)^2+(\beta_1-\beta_2)^2+(y_1-y_2)^2\right\}\right]^{\frac{1}{2}} \tag{5}$$

2.4 Score function [8].

$$\mathrm{Sc}(e_1)=\frac{2+x_1-\beta_1-y_1}{3} \tag{6}$$

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2.5 Rough neutrosophic set [48]

Assume that $\ddot{U}$ *is* a nonvoid set. Assume that $\ddot{R}$ is an equivalence relation on $\ddot{U}$. Let $\ddot{F}$ be an NS in $\ddot{U}$ with the truth Membership Function (MF) $\ddot{\mu}_{\ddot{F}}(\ddot{\theta}),$, indeterminacy MF $\ddot{\nu}_{\ddot{F}}(\ddot{\theta})$, and falsity MF $\ddot{\omega}_{\ddot{F}}(\ddot{\theta})$. The lower and the upper approximations of $\ddot{F}$ in the approximation ($\ddot{U}$, $\ddot{R}$) presented by $\underline{\ddot{N}}(\ddot{F})$ and $\overline{\ddot{N}}(\ddot{F})$ are presented as:

$$\underline{\ddot{N}}(\ddot{F})=\left\langle<\ddot{\theta},\mu_{\underline{\ddot{N}}(\ddot{F})}(\ddot{\theta}),\nu_{\underline{\ddot{N}}(\ddot{F})}(\ddot{\theta}),\omega_{\underline{\ddot{N}}(\ddot{F})}(\ddot{\theta})>/\ \ddot{y}\in[\ddot{\theta}]_{\ddot{R}},\ddot{\theta}\in\ddot{U}\right\rangle \tag{7}$$

$$\overline{\ddot{N}}(\ddot{F})=\left\langle<\ddot{\theta},\mu_{\overline{\ddot{N}}(\ddot{F})}(\ddot{\theta}),\nu_{\overline{\ddot{N}}(\ddot{F})}(\ddot{\theta}),\omega_{\overline{\ddot{N}}(\ddot{F})}(\ddot{\theta})>/\ \ddot{y}\in[\ddot{\theta}]_{\ddot{R}},\ddot{\theta}\in\ddot{U}\right\rangle \tag{8}$$

where $\mu_{\underline{\ddot{N}}(\ddot{F})}(\ddot{\theta})=\underset{\ddot{y}\in[\ddot{\theta}]_{\ddot{R}}}{\wedge}\mu_{\ddot{F}}(\ddot{y}),\nu_{\underline{\ddot{N}}(\ddot{F})}(\ddot{\theta})=\underset{\ddot{y}\in[\ddot{\theta}]_{\ddot{R}}}{\vee}\nu_{\ddot{F}}(\ddot{y}),\omega_{\underline{\ddot{N}}(\ddot{F})}(\ddot{\theta})=\underset{\ddot{y}\in[\ddot{\theta}]_{\ddot{R}}}{\vee}\omega_{\ddot{F}}(\ddot{y})$

$\mu_{\overline{\ddot{N}}(\ddot{F})}(\ddot{\theta})=\underset{\ddot{y}\in[\ddot{\theta}]_{\ddot{R}}}{\vee}\mu_{\overline{\ddot{N}}(\ddot{F})}(\ddot{y}),\nu_{\overline{\ddot{N}}(\ddot{F})}(\ddot{\theta})=\underset{\ddot{y}\in[\ddot{\theta}]_{\ddot{R}}}{\wedge}\nu_{\ddot{F}}(\ddot{y}),\omega_{\overline{\ddot{N}}(\ddot{F}))}(\ddot{\theta})=\underset{\ddot{y}\in[\ddot{\theta}]_{\ddot{R}}}{\wedge}\omega_{\ddot{F}}(\ddot{y})$

So, $\ddot{U},\ddot{R}$

$0\le\mu_{\overline{\ddot{N}}(\ddot{F})}(\ddot{\theta})+\nu_{\overline{\ddot{N}}(\ddot{F})}(\ddot{\theta})+\omega_{\overline{\ddot{N}}(\ddot{F}))}(\ddot{\theta}))\le 3.$

Here, $\vee$ and $\wedge$ present respectively the max and "min'' operator. $\mu_{\ddot{F}}(\ddot{y}),\nu_{\ddot{F}}(\ddot{y}),$,and $\omega_{(\ddot{F})}(\ddot{y})$ are the truth MF, indeterminacy MF, and falsity MF of $\ddot{y}$ w.r.t. $\ddot{F}$. Here, $\underline{\ddot{N}}(\ddot{F})$ and $\overline{\ddot{N}}(\ddot{F})$ are NSs in $\ddot{U}$.

The NS mapping $\overline{\ddot{N}},\underline{\ddot{N}}:N(\ddot{U}\to N(\ddot{U}))$ denote as the lower and upper RNS approximation operators. The pair $(\underline{\ddot{N}}(\ddot{F}),\overline{\ddot{N}}(\ddot{F}))$ is called the RNS in ($\ddot{U},\ddot{R}$).

RNN-ARAS strategy in RNN setting

Step 1. Formulate the decision matrix

Decision matrix $\ddot{D}$ is formulated using the RNN rating values of the alternatives provided by the decision maker (DM).

$$\ddot{D}=\left\langle\underline{\ddot{y}}_{ij},\overline{\ddot{y}}_{ij}\right\rangle_{m\times n}=$$

	C_1'''	C_2'''	$\cdots$	C_n'''
A_1'''	$\left\langle\underline{\ddot{y}}_{11},\overline{\ddot{y}}_{11}\right\rangle$	$\left\langle\underline{\ddot{y}}_{12},\overline{\ddot{y}}_{12}\right\rangle$	$\cdots$	$\left\langle\underline{\ddot{y}}_{1n},\overline{\ddot{y}}_{1n}\right\rangle$
A_2'''	$\left\langle\underline{\ddot{y}}_{21},\overline{\ddot{y}}_{21}\right\rangle$	$\left\langle\underline{\ddot{y}}_{22},\overline{\ddot{y}}_{22}\right\rangle$	$\cdots$	$\left\langle\underline{\ddot{y}}_{2n},\overline{\ddot{y}}_{2n}\right\rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_m'''	$\left\langle\underline{\ddot{y}}_{m1},\overline{\ddot{y}}_{m1}\right\rangle$	$\left\langle\underline{\ddot{y}}_{22},\overline{\ddot{y}}_{22}\right\rangle$	$\cdots$	$\left\langle\underline{\ddot{y}}_{mn},\overline{\ddot{y}}_{mn}\right\rangle$

(9)

Here, $\left\langle \underline{\ddot{y}}_{ij}, \overline{\ddot{y}}_{ij} \right\rangle_{m\times n} = \left\langle \left\langle \underline{\tau}_{ij}, \underline{\iota}_{ij}, \underline{\phi}_{ij} \right\rangle, \left\langle \overline{\tau}_{ij}, \overline{\iota}_{ij}, \overline{\phi}_{ij} \right\rangle \right\rangle$ denotes the RNN rating value of A_i''' w.r.t. c_j''' provided by the DM.

Step 2. Convert the decision matrix into neutrosophic decision matrix using the Accumulated Geometric Operator (AGO).

We convert the RNN to SVNN by the AGO [57] as follows:

$$\begin{aligned}\left\langle \underline{\ddot{y}}_{ij}, \overline{\ddot{y}}_{ij} \right\rangle_{AGO} &= \left\langle \left\langle \underline{\tau}_{ij}, \underline{\iota}_{ij}, \underline{\phi}_{ij} \right\rangle, \left\langle \overline{\tau}_{ij}, \overline{\iota}_{ij}, \overline{\phi}_{ij} \right\rangle \right\rangle_{AG0} \\ &= \left\langle \left(\underline{\tau}_{ij}.\overline{\tau}_{ij} \right)^{0.5}, \left(\underline{\iota}_{ij}.\overline{\iota}_{ij} \right)^{0.5}, \left(\underline{\phi}_{ij}.\overline{\phi}_{ij} \right)^{0.5} \right\rangle \\ &= \left\langle \tau_{ij}', \iota_{ij}', \phi_{ij}' \right\rangle\end{aligned} \tag{10}$$

The decision matrix is transformed to neutrosophic decision matrix $d_{\tilde{N}}$

$$\ddot{d}_{\tilde{N}} = \left\langle \tau_{ij}', \iota_{ij}', \phi_{ij}' \right\rangle = \left\langle \phi_{ij}'\tau_{ij}', 1-\iota_{ij}', \tau_{ij}' \right\rangle$$

$$= \begin{array}{c|cccc} & C_1''' & C_2''' & \ldots & C_n''' \\ \hline A_1''' & \left\langle \tau_{11}', \iota_{11}', \phi_{11}' \right\rangle & \left\langle \tau_{12}', \iota_{12}', \phi_{12}' \right\rangle & \ldots & \left\langle \tau_{1n}', \iota_{1n}', \phi_{1n}' \right\rangle \\ A_2''' & \left\langle \tau_{21}', \iota_{21}', \phi_{21}' \right\rangle & \left\langle \tau_{22}', \iota_{22}', \phi_{22}' \right\rangle & \ldots & \left\langle \tau_{2n}', \iota_{2n}', \phi_{2n}' \right\rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \\ & & & & \ldots \\ . & \ldots & \ldots & \ldots & \\ A_m''' & \left\langle \tau_{m1}', \iota_{m1}', \phi_{m1}' \right\rangle & \left\langle \tau_{m2}', \iota_{m2}', \phi_{m2}' \right\rangle & \ldots & \left\langle \tau_{mn}', \iota_{mn}', \phi_{mn}' \right\rangle \end{array} \tag{11}$$

Step 3. Standardize the decision matrix

Since criteria are two types, namely, benefit or cost, then there is a need to standardize them using the following formula [28]

$$\ddot{D}_{ij} = \begin{cases} \left\langle \left\langle \tau_{ij}', \iota_{ij}', \right\rangle \right\rangle, C_j''' \text{ is a benefit critrion} \\ \left\langle \ddot{\varphi}_{ij}', 1-\ddot{\iota}_{ij}', \ddot{\tau}_{ij}' \right\rangle, C_j''' \text{ is a cost critrion} \end{cases} \tag{12}$$

Then the standardized decision matrix appears of the form:

$$\ddot{d}_{\tilde{N}} = \left\langle \ddot{\tau}_{ij}'', \ddot{\iota}_{ij}'' \ddot{\varphi}_{ij}'' \right\rangle_{m\times n} \tag{13}$$

Step 4 . Formulate the weighted decision matrix

$$Y_{ij} = \left\langle \ddot{\tau}_{m1}''', \ddot{\iota}_{m1}''' \ddot{\varphi}_{m1}''' \right\rangle = \omega_j''' \otimes \left\langle \ddot{\tau}_{ij}'', \ddot{\iota}_{ij}'' \ddot{\varphi}_{ij}'' \right\rangle = \left(1-\left(1-\ddot{\tau}_{ij}''\right)^{\omega_j'''}, \ddot{\iota}_{ij}''^{\omega_j'''}, \ddot{\varphi}_{ij}''^{\omega_j'''} \right). \tag{14}$$

Step 5 ***Determine the optimal function values and deneutrosophication***

To calculate the optimal functions values of the weighted decision matrix, we can use the equation (15).

$$\begin{aligned}\Theta_r &= \left\langle \ddot{\tau}_{r1}''', \ddot{\iota}_{r1}''' \ddot{\varphi}_{r1}''' \right\rangle \oplus \left\langle \ddot{\tau}_{r2}''', \ddot{\iota}_{r2}''' \ddot{\varphi}_{r2}''' \right\rangle \oplus \ldots \oplus \left\langle \ddot{\tau}_{rn}''', \ddot{\iota}_{rn}''' \ddot{\varphi}_{rn}''' \right\rangle \\ &= \left\langle 1-\prod_{s=1}^{n}(1-\ddot{\tau}_{rs}'''), \prod_{s=1}^{n}(\ddot{\iota}_{rs}'''), \prod_{s=1}^{n}(\ddot{\varphi}_{rs}''') \right\rangle \end{aligned} \quad \text{where r = 1, 2, …, m} \tag{15}$$

Deneutrosophication

We calculate the score values of the elements of (15) using the formula of score function [8]

$$\mathrm{Sc}(\Theta_r) = \frac{2 + \ddot{\tau}''_{rs} - \ddot{\iota}''_{rs} - \ddot{\varphi}''_{rs}}{3}$$

Step 6: **Evaluate the Alternative Utility Degree (AUD*)***

The AUD $\mathrm{Sc}(\Theta_r)$ with the best suited $Sc(\Theta^{\bullet})$ is shown in the formula (15). The alternative's utility degree of Ω_r is presented as follows:

$$\Omega_r = \frac{Sc(\Theta_r)}{Sc(\Theta^{\bullet})}; r = 1,2,...,m. \qquad (15)$$

Here, $Sc(\Theta^{\bullet}) = Max\{Sc(\Theta_1), Sc(\Theta_2), ..., Sc(\Theta_n)\}$

Step 7: Rank the alternatives

The descending order of can be used to identify the relative priority of workable alternatives Ω_r. That is the alternative with the highest value of Ω_r is the best choice.

Illustrative Example

We consider the following problem [59]. Assume that an expert intends to buy the best suitable smartphone from the initial selected smartphones ($\ddot{A}_1, \ddot{A}_2, \ddot{A}_3$. The attributes are:

I. Features $\ddot{C}_1$,

II. price $\ddot{C}_2$,

III. customer support $\ddot{C}_3$ and

IV. risk factor $\ddot{C}_4$.

Weights of the four attributes are considered as 0.3, .03, 0.3, 0.1 respectively. Based on the developed RNN-COPRAS strategy, the problem is solved as follows:

Step 1.

The RNN decision matrix (see the Table 1) is formulated based on rating value of alternative over the criterion.

Table 1. RNN decision matrix

	$\ddot{C}_1$	$\ddot{C}_2$	$\ddot{C}_3$	$\ddot{C}_4$
	benefit type	cos*t type*	*benefit type*	cos*t type*
$\ddot{A}_1$	$\langle(.6,.3,.3),(0.8,0.1,0.1)\rangle$	$\langle(.6,.4,.4),(0.8,0.2,0.2)\rangle$	$\langle(.6,.4,.4),(0.8,0.2,0.4)\rangle$	$\langle(.7,.4,.7),(0.9,0.2,0.1)\rangle$
$\ddot{A}_2$	$\langle(0.7,0.3,0.3),(0.9,0.1,0.3)\rangle$	$\langle(0.6,0.3,0.3),(0.8,0.3,0.3)\rangle$	$\langle(0.6,0.2,0.2),(0.8,0.4,0.2)\rangle$	$\langle(0.7,0.3,0.2),(0.9,0.3,0.3)\rangle$
$\ddot{A}_2$	$\langle(0.6,0.2,0.2),(0.8,0.0,0.2)\rangle$	$\langle(0.7,0.3,0.2),(0.9,0.1,0.1)\rangle$	$\langle(0.7,0.4,0.6),(0.9,0.2,0.4)\rangle$	$\langle(0.6,0.3,0.2),(0.8,0.1,0.1)\rangle$

Step 2.

Using the formula (10), RNN decision matrix is converted to SVNN decision matrix.

Table 2. SVNN decision matrix

	$\ddot{C}_1$	$\ddot{C}_2$	$\ddot{C}_3$	$\ddot{C}_4$
	benefit type	*cost type*	*benefit type*	*cost type*
$\ddot{A}_1$	⟨0.69282, 0.1732051, 0.173205⟩	⟨0.69282, 0.282843, 0.282843⟩	⟨0.69282, 0.282843, 0.4⟩	⟨0.793725, 0.282843, 0.264575⟩
$\ddot{A}_2$	⟨0.793725, 0.1732051, 0.244949⟩	⟨0.69282, 0.3, 0.3⟩	⟨0.69282, 0.282843, 0.2⟩	⟨0.793725, 0.3, 0.244949⟩
$\ddot{A}_3$	⟨0.69282, 0, 0.2⟩	⟨0.793725, 0.173205, 0.141421⟩	⟨0.793725, 0.282843, 0.489898⟩	⟨0.69282, 0.173205, 0.141421⟩

Step 3.

The SVNN decision matrix is standardized (see Table 3) using the formula (12)

Table 3. Standardized decision matrix

	$\dot{C}_1$	$\dot{C}_2$	$\dot{C}_3$	$\dot{C}_4$
	benefit type	*cost type*	*benefit type*	*cost type*
$\dot{A}_1$	⟨0.69282, 0.1732051, 0.173205⟩	⟨0.282843 0.717157 0.69282⟩	⟨0.69282, 0.282843, 0.4⟩	⟨0.264575 0.717157 0.793725⟩
$\dot{A}_2$	⟨0.793725, 0.1732051, 0.244949⟩	⟨0.3 0.7 0.69282⟩	⟨0.69282, 0.282843, 0.2⟩	⟨0.264575 0.717157 0.793725⟩
$\dot{A}_3$	⟨0.69282, 0, 0.2⟩	⟨0.141421 0.826795 0.793725⟩	⟨0.793725, 0.282843, 0.489898⟩	⟨0.141421 0.826795 0.69282⟩

Step 4.

Using the formula (12), and standardized matrix, the weighted decision matrix is formulated (see table 4).

Table 4. Weighted decision matrix

	$\dot{C}_1$	$\dot{C}_2$	$\dot{C}_3$	$\dot{C}_4$
	benefit type	*cost type*	*benefit type*	*cost type*
$\dot{A}$	⟨0.298192922 0.590974 0.590974⟩	⟨0.094925509 0.905074 0.895749⟩	⟨0.298193 0.684642 0.759658⟩	⟨0.030263 0.967301 0.977163⟩
$\dot{A}_2$	⟨0.377221329 0.590974 0.655726⟩	⟨0.101476558 0.898523 0.895749⟩	⟨0.298193 0.684642 0.617034⟩	⟨0.027706 0.964961 0.977163⟩
$\dot{A}_3$	⟨0.298192922 0 0.617034⟩	⟨0.044712655 0.944538 0.933042⟩	⟨0.377221 0.684642 0.807294⟩	⟨0.015132 0.98116 0.963967⟩

Step 5. Determine the optimal function values and deneutrosophication

To calculate the optimal functions values of the weighted decision matrix, we employ the equation (13).

$$\Theta_r = \left\langle \dddot{\tau}_{r1}^{'''}, \dddot{\imath}_{r1}^{'''} \dddot{\varphi}_{r1}^{'''} \right\rangle \oplus \left\langle \dddot{\tau}_{r2}^{'''}, \dddot{\imath}_{r2}^{'''} \dddot{\varphi}_{r2}^{'''} \right\rangle \oplus \ldots \oplus \left\langle \dddot{\tau}_{rn}^{'''}, \dddot{\imath}_{rn}^{'''} \dddot{\varphi}_{rn}^{'''} \right\rangle$$
$$= \left\langle 1 - \prod_{s=1}^{n} (1 - \dddot{\tau}_{rs}^{'''}), \prod_{s=1}^{n} (\dddot{\imath}_{rs}^{'''}), \prod_{s=1}^{n} (\dddot{\varphi}_{rs}^{'''}) \right\rangle \quad \text{where r = 1, 2, …, m} \qquad (16)$$

The obtained optimal function values are shown in table 5.

Table 5. Optimal function values

Alternative	Optimal functional value
A_1	(0.567711539, 0.354224, 0.392952)
A_2	(0.618162554, 0.350809, 0.354148)
A_3	(0.588790152, 0, 0.448027)

The score values of the alternatives are shown in Table 6**.**

Table6. Score values of the alternatives

Alternatives	$Sc(\Theta_1)$	$Sc(\Theta_2)$	$Sc(\Theta_3)$
Values	0.606845	0.637735	0.713588

$$Sc(\Theta^{\bullet}) = Max\{0.606845, 0.637735, 0.713588\}$$
$$= 0.713588$$

Step 6: Evaluate the Alternative Utility Degree (AUD)

Using the formula (15), the alternative's utility degree of Ω_r is obtained as follows:

$\Omega_1 = 0.85041, \quad \Omega_2 = 0.8937, \ \Omega_3 = 1$ (17)

Step 7: Rank the alternatives

Since $\Omega_3 > \Omega_2 > \Omega_1$, we have $\ddot{A}_3 \succ \ddot{A}_2 \succ \ddot{A}_1$.

So, $\ddot{A}_3$ is the best alternative.

Comparison The problem is solved with RNN-MABAC and RNN-COPRAS we obtain the same ranking order (see Table 5).

Table 5. Comparison Table

Strategy	Ranking order	Best alternative
RNN-ARAS [proposed]	$\ddot{A}_3 \succ \ddot{A}_2 \succ \ddot{A}_1$	$\ddot{A}_3$
RNN-COPRAS [59]	$\ddot{A}_3 \succ \ddot{A}_2 \succ \ddot{A}_1$	$\ddot{A}_3$
RNN-MABAC [58]	$\ddot{A}_3 \succ \ddot{A}_2 \succ \ddot{A}_1$	$\ddot{A}_3$

Conclusion

In this paper, the RNN-ARAS strategy in the RNN environment has developed. An illustrative example of an investment problem is solved to reflect the effectiveness of the proposed RNN-ARAS strategy. The developed RNN-ARAS strategy can be effectively used to solve real-world MADM problems with inconsistent and incomplete information. We hope that this paper will inspire researchers to conduct research in the field of MADM. The developed RNN- ARAS strategy can be explored for group decision-making strategy using a suitable aggregation operator which we shall do in the future.

Future Research Directions

In the future, the developed RNN-ARAS can be used to solve other MADM problems such as brick selection [65, 66], weaver selection [67, 68], teacher selection [69, 70, 71], school choice [72], supply chain [73], green supplier selection [74], healthcare [75], etc.

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